

# THREE STEPS TO FIND COMPLETE PORTFOLIO

## Step 1 : Specify the return characteristics of all securities

- Specify the return characteristics of all securities (expected returns, variances, covariances)
- The first thing in your assignment is to construct the table below as it gives you the returns, risk and correlation coefficient (CC) of your securities and to get this table you basically have to use the Air New Zealand example, this table input is step number one
- **Note:** Excel sample is provided as your template, please use assignment 1 excel which contains all the calculations guide you need to complete your portfolio excel calculations. Air Zealand is your guide too.

### Portfolio Risk and Return

#### ■ Example with debt and equity

|                 | Debt | Equity |
|-----------------|------|--------|
| Expected Return | 8%   | 13%    |
| S.D.            | 12%  | 20%    |
| Correlation     | 0.3  |        |

$$E(r_p) = 8W_D + 13W_E$$

$$\sigma_p^2 = 12^2 W_D^2 + 20^2 W_E^2 + 2 * 12 * 20 * .3 * W_D * W_E$$

- We need to find proper weights to get minimum risk portfolio (minimum-variance portfolio)

## Step 2 : Calculate the properties of optimal portfolio (P)

Establish the risky portfolio

- Determine the weights within the optimal risky portfolio
- Calculate the properties of P using the weights determined in previous step

Once you know the returns, risk and CC of the securities, now step no 2 is to find P and to find P you need to calculate the weights of the two securities optimal risk portfolio and once you find the weights together with the information in the table in step 1 above then you can calculate the properties (risk and return) of Optimal risk portfolio. **So step two is to find optimal risk portfolio**

- Specify the return characteristics of all securities (expected returns, variances, covariances)
  - Establish the risky portfolio
    - Determine the weights within the optimal risky portfolio
    - Calculate the properties of P using the weights determined in previous step
- 

### Step 3: Find complete portfolio

Allocate funds between the optimal risky portfolio and the risk-free asset

- Calculate the fraction of the complete portfolio allocated to portfolio P and to T-bills
- Calculate the share of the complete portfolio invested in each asset and in T-bills

To find complete portfolio you need to calculate the weights that you invest in optimal risk portfolio, that is Y and then once you know the weights and you know the returns and risk of the two assets then you can calculate the return and risk of the complete portfolio, that is the final product

Build two portfolios, two stock portfolios with risk free asset

- 1- S&P 500 and one NASDAQ Company 1
- 2- S&P 500 and one NASDAQ Company 2
- 3- Risk free rate

Choose a better portfolio which is optimal

- Do questionnaire on the test book to test your degree of risk aversion, how much risk you would like to take and with that information then you can find your complete portfolio which is your final product. Before start building portfolio please spend a lot of times to select the right stocks. Sample period is very important which is from = ***The annual risk free rate is 2% and the sample period is from 1 June 2012 to 4 June 2019 (Please use these exact days for sample period selection).*** Please put exact date in yahoo finance to collect the data, note some companies are cross listed i.e that is the same company as S&P 500 and NASDAQ Company, its ok as long as it is listed as S&P 500 it doesn't matter. The written report assignment 3 to 5 pages is basically the summary of your calculations and some analysis and the excel shows all your calculations. Help and hints on the bottom where it tell you where to collect the data
- Equal probability if you add them together and divide by the number of observation is called arithmetic average

## Expected Returns and the Arithmetic Average (assuming the equal probability)

$$E(r) = \sum_{s=1}^n p(s)r(s) = \frac{1}{n} \sum_{s=1}^n r(s)$$

and

alternatively we have geometric average, so the geometric average instead of adding them together and divide by n, we multiply them together using 1 plus each return and multiply all of them together that is equal to 1 plus the geometric average to the power of n

$$TV = (1+r_1) * (1+r_2) * \dots * (1+r_n) = (1+g)^n$$

TV = Terminal Value of the Investment

$$g = TV^{1/n} - 1$$

g = geometric average rate of return

- In assignment we mainly use arithmetic average because it is used to focus future returns and calculating past returns we use geometric average

- Geometric mean considers the principle of compounding, and we assume that dividends are reinvested
- Arithmetic mean will always be greater than or equal to the geometric mean

$$r_G \approx r_A - \frac{1}{2} \sigma^2$$

and

arithmetic average is always higher than the geometric average or at least equal to geometric. Note whenever you calculate standard deviation please use n-1,

- Variance = expected value of squared deviations (assuming equal probability)

$$\sigma^2 = \frac{1}{n} \sum_{s=1}^n [r(s) - \bar{r}]^2$$

- When we deal with a sample instead of the population, in order to eliminate the bias, we use (n-1) instead of n in the above formula
- Standard deviation is the square root of variance

use historical data in yahoo finance to calculate returns, then select the time period, the frequency choose monthly date, then apply, download data, avoid week end, use adjusted closing price, it takes care if dividend and split price.

- Calculating air new zealand return use two formulas 1=holding period return formula  $P1-P0$  DIVE BY  $P0$  and 2=Log return, but use both but for log(Geometric) return make sure you get the time right, you lose one price

| 1  | Date       | Open  | High  | Low   | Close | Adj Close | Volume          | Monthly return 1=(P1-P0)/P0 | Monthly return 2=(P1-P0)/P0 |
|----|------------|-------|-------|-------|-------|-----------|-----------------|-----------------------------|-----------------------------|
| 2  | 31/05/2011 | 1.09  | 1.16  | 1.01  | 1.12  | 0.641019  | 16683856        |                             |                             |
| 3  | 30/06/2011 | 1.12  | 1.2   | 1.11  | 1.17  | 0.669636  | 9837098         | 0.0437                      | 0.0446                      |
| 4  | 31/07/2011 | 1.17  | 1.18  | 1.02  | 1.11  | 0.635296  | 7852973         | -0.0526                     | -0.0513                     |
| 5  | 31/08/2011 | 1.11  | 1.14  | 1.07  | 1.09  | 0.623849  | 7826644         | -0.0182                     | -0.0180                     |
| 6  | 30/09/2011 | 1.08  | 1.09  | 1.01  | 1.05  | 0.614676  | 13830136        | -0.0148                     | -0.0147                     |
| 7  | 31/10/2011 | 1.05  | 1.07  | 0.955 | 0.97  | 0.567843  | 5080148         | -0.0793                     | -0.0762                     |
| 8  | 30/11/2011 | 0.98  | 0.98  | 0.85  | 0.9   | 0.526865  | 5811209         | -0.0749                     | -0.0722                     |
| 9  | 31/12/2011 | 0.905 | 0.91  | 0.85  | 0.91  | 0.532719  | 4925011         | 0.0110                      | 0.0111                      |
| 10 | 31/01/2012 | 0.92  | 0.935 | 0.85  | 0.865 | 0.506376  | 5582355         | -0.0507                     | -0.0495                     |
| 11 | 29/02/2012 | 0.87  | 0.89  | 0.85  | 0.85  | 0.497595  | 9031018         | -0.0175                     | -0.0173                     |
| 12 | 31/03/2012 | 0.85  | 0.89  | 0.85  | 0.89  | 0.545672  | 3809630         | 0.0922                      | 0.0966                      |
| 13 | 30/04/2012 | 0.89  | 0.94  | 0.87  | 0.87  | 0.533409  | 5141998         | -0.0227                     | -0.0225                     |
| 14 | 31/05/2012 | 0.87  | 0.89  | 0.84  | 0.86  | 0.527278  | 4244830         | -0.0116                     | -0.0115                     |
| 15 | 30/06/2012 | 0.875 | 0.92  | 0.86  | 0.9   | 0.551803  | 3612998         | 0.0455                      | 0.0465                      |
| 16 | 31/07/2012 | 0.9   | 1.11  | 0.895 | 1.09  | 0.668294  | 6711228         | 0.1915                      | 0.2111                      |
| 17 | 31/08/2012 | 1.09  | 1.18  | 1.06  | 1.16  | 0.711212  | 27783364        | 0.0622                      | 0.0642                      |
| 18 | 30/09/2012 | 1.165 | 1.285 | 1.16  | 1.24  | 0.784562  | 18729285        | 0.0982                      | 0.1031                      |
| 19 | 31/10/2012 | 1.24  | 1.295 | 1.18  | 1.28  | 0.80987   | 40194085        | 0.0317                      | 0.0323                      |
| 20 | 30/11/2012 | 1.29  | 1.38  | 1.275 | 1.295 | 0.819361  | 19274996        | 0.0117                      | 0.0117                      |
| 21 | 31/12/2012 | 1.295 | 1.32  | 1.235 | 1.25  | 0.790889  | 19857917        | -0.0354                     | -0.0347                     |
| 22 | 31/01/2013 | 1.25  | 1.41  | 1.24  | 1.4   | 0.885796  | 26456411        | 0.1133                      | 0.1200                      |
| 23 | 28/02/2013 | 1.4   | 1.54  | 1.38  | 1.535 | 0.971212  | 24638247        | 0.0921                      | 0.0964                      |
| 24 | 31/03/2013 | 1.535 | 1.535 | 1.425 | 1.5   | 0.968635  | 23952917        | -0.0027                     | -0.0027                     |
| 25 | 30/04/2013 | 1.495 | 1.545 | 1.45  | 1.465 | 0.946034  | 80355043        | -0.0236                     | -0.0233                     |
| 85 | 30/04/2018 | 3.27  | 3.39  | 3.13  | 3.13  | 3.13      | 31687222        | -0.0438                     | -0.0428                     |
| 86 | 31/05/2018 | 3.15  | 3.28  | 3.15  | 3.15  | 3.15      | 6547138         | 0.0190                      | 0.0192                      |
| 87 |            |       |       |       |       |           | Monthly average | 0.0191                      | 0.0222                      |
| 88 |            |       |       |       |       |           | Monthly S.D.    | 0.0760                      | 0.0780                      |
| 89 |            |       |       |       |       |           | Annual return   | 0.2549                      | 0.3015                      |
| 90 |            |       |       |       |       |           | Annual S.D.     | 0.2634                      | 0.2703                      |

Note: In your assignment 1, you can use either method to calculate your stock or index returns to get full marks. If you click on certain cells, you can see the formula that I use to do the calculation. You can use them to do your assignment 1.

91 However, you will find that return 2 is always higher than return 1.

Air NZ

Ready Average: 2.06529412 Count: 85 Sum: 175.635

|                 |        |        |
|-----------------|--------|--------|
| Monthly average | 0.0191 | 0.0222 |
| Monthly S.D.    | 0.0760 | 0.0780 |
| Annual return   | 0.2549 | 0.3015 |
| Annual S.D.     | 0.2634 | 0.2701 |

Note: In your assignment 1, you can use either method to calculate your stock or index returns to get full marks. If you click on certain cells, you can see the formula that I use to do the calculation. You can use them to do your assignment 1. However, you will find that return 2 is always higher than return 1.

- Then use excel calculation to calculate the 1- monthly average return 2- Monthly SD 3- Annual return 4- Annual SD
- In excel on stream column H=geometric and column I=Arithmetic return

### ■ How to calculate the return and standard deviation (S.D.) for your stock?

### ■ Price — returns — mean of returns — S.D.

■  $r = (P_{t+1} - P_t) / P_t$  (HPR) or  $r = \ln (P_{t+1} / P_t)$

### ■ Arithmetic Mean Return

$$\bar{r} = \frac{1}{n} \sum_{i=1}^n r_i$$

### ■ S.D.

$$Variance = \sum (r - \bar{r})^2 / (n - 1)$$

$$S.D. = \sqrt{Variance}$$

- Note when we calculate returns we divide by n but when we calculate variance or correlation (Correlation coefficient) we divide by n-1
- In order to choose stocks you probably need to look at the risks and returns of look at 8 to 10 stocks, you need to choose 3 stocks you like, when you look at return, first I have given you the annual risk free rate which is 2 % and if your stock annual return is below 2% that means the stock is not very good, look for stock that is higher than the market return also compare the return with the index returns(index returns shows the average of the whole market), compare the return of stocks with other stocks, compare the return of stock 1,2,3,4 etc
- In terms of risk, you would like the risk to be lower and compare the risk of stock with the risk of other stocks e.g compare the SD of stock 1, 2, 3, etc
- After you calculate all the monthly returns then you need to calculate average return and also in excel you can calculate standard deviation, so the input of calculation SD IS JUST THE MONTHLY returns, basically all the information about returns and risks comes from prices. When you calculate the monthly returns and risk then you need to convert the numbers to annual numbers. To calculate annual return we is  $(1+rm)^{12}-1$  one plus monthly return to the power of twelve minus one. Then you need to calculate the annual SD= monthly SD multiply by square root of 12=  $SDm \times 12^{.5}$
- When you choose asset onto a portfolio you consider three things i.e that when you combine or include assets into the existing portfolio 1- risk 2- return 3- correlations, the

correlations is the one that changes the final answer

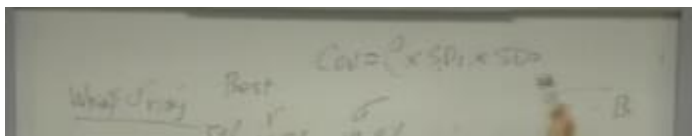
**Rule 5:** When two risky assets with variances  $\sigma_1^2$  and  $\sigma_2^2$ , respectively, are combined into a portfolio with portfolio weights  $w_1$  and  $w_2$ , respectively, the portfolio variance is given by:

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \text{Cov}(r_1, r_2)$$

- To calculate the covariance if you know the returns of the two assets, use the long formula and if it is equal probability you add them together and divide by n-1 but if unequal probability then you multiply by the probability
- 

$$\text{Cov}(A, B) = \sum_s \text{Pr}(s) [r_A(s) - E(r_A)] [r_B(s) - E(r_B)]$$

- 
- Another to calculate covariance is covariance equal to correlation coefficient times SD1 and times SD2 i.e.  $\text{cov} = \text{correlation coef} \times \text{SD1} \times \text{SD2}$
- 



A photograph of a whiteboard with handwritten text. The main formula is  $\text{Cov} = r \times \text{SD1} \times \text{SD2}$ . There are also some other handwritten notes like 'What if r = -0.86' and 'Best'.

- Correlation coefficient is an easier statistics to interpret than the covariance
- Correlation coefficient changes from -1 to 1

$$\rho(\text{Best}, \text{Sugarcane}) = \frac{\text{Cov}[r_{\text{best}}, r_{\text{sugarcane}}]}{\sigma_{\text{Best}} \sigma_{\text{Sugarcane}}} = \frac{-240.5}{18.9 * 14.73} = -.86$$

- This large negative correlation (close to -1) confirms the strong tendency of Best and Sugarcane stocks to move inversely

- 
- Because you have to choose three stocks for your portfolio then you need to think of their connections too .i.e good diversification apart from higher return and lower risk, in theory we want to good diversification effect then we tend choose stocks with negative

diversification but in real life its not easy most stock will have positive diversification e.g .4 below is still good

- Also think about industry when choosing your stock, don't choose all the stocks in the same industry, something happens in the industry affect all the stocks in your portfolio if they are from the same industry
- So, when choosing stocks look at risk and return, correlation coefficient and try to diversify among industries
- Negative correlation coefficient (cc) effect portfolio returns and risk, cc reduces portfolio risk and does not affect portfolio returns i.e returns remain unchanged. The reason is because when we calculate portfolio returns we only use weighted average, there is no cc in the equation and the return are mean or average returns
- Three things affect or impact utility 1- risk 2- return 3-risk aversion of individual investors
- **Risk averse = Positive A**
- Risk neutral= A is zero
- Risk seeking= A is negative
- Elements that affect risk aversion, age, financial situation, personality, life stage
- No formula to calculate A, in your assignment you need to use the questionnaire Risk seeking and decide the A level, if you are quite very risk averse your A level will be 5-6, if you are medium averse 3-4 and if you not very sensitive about the risk you are 1-2
- P is optimal risk portfolio along the CAL and the final complete product is called C, the risk free asset (rf) on the graph and its risk is zero then you have P (Optimal risk portfolio) on the graph but you need to find C. The connection between rf and P is called CAL because this decision is capital allocation decision and the final product is called complete portfolio C. Optimal portfolio An [efficient portfolio](#) most preferred by an [investor](#) because its [risk](#)/reward characteristics approximate the [investor's](#) utility function. A [portfolio](#) that maximizes an investor's preferences with respect to [return](#) and [risk](#). Optimal portfolio does not include risk free asset. Complete portfolio is the *entire [portfolio](#), including [risky](#) and [risk-free assets](#)*. Where C is on the CAP depends on the degree of risk Aversion of an investor and to make capital allocation decision there are two things .i.e 1- is the technical part that is to find CAL and to find CAL you need to find the return and risk information for the risk assets and the optimal portfolio (P), once you know the risk and returns of the two then you can draw the line for CAL and is fixed but where exactly the point C is depends on the personal decision determined the degree of risk aversion. To decide the point where P is you need to find the variable Y because Y is the % you invest in P ( the % you put in P is Y) and 1-Y is the % you put in free assets, if Y is large that if you put more money in P then the C will be to the right Hand side and if you put more money in risk free assets the P will move to the left hand side, so if we know the value of Y then we know where C is

- To calculate the return of C is

$$E(r_c) = yE(r_p) + (1 - y)r_f$$

**$r_c$  = complete or combined portfolio**

**For example,  $y = .75$**

$$\begin{aligned} E(r_c) &= .75(15) + .25(7) \\ &= 13\% \end{aligned}$$

**Since  $\sigma_{r_f} = 0$ , then**

$$\sigma_c = y \sigma_p^*$$

**\* Rule 4 in Topic 1**

- What is the risk of C
- Any where on the left hand side of P is lending and anywhere on the right hand side of P is borrowing
- How to find C we need 3 equations, rule no 3, rule no 4 then utility function
- In order to calculate Y , we need information about return of the optimal risk portfolio, return of risk free rate, the risk of optimal risk portfolio but you need to decide A ( the degree of risk aversion i.e how you don't like risk and importunately you can not calculate A, you need to make judgement of the value of A (1-2, 3-4 and 5-6)

- In our example, for an investor with risk aversion of 4

$$y^* = \frac{15 - 7}{.01 * 4 * 22^2} = .41$$

- It means that the complete portfolio is to invest 41% in risky assets and 59% in risk-free asset
- Return, risk & utility of portfolio

$$E(r_c) = 7 + [.41 * (15 - 7)] = 10.28\%$$

$$\sigma_c = .41 * .22 = 9.02\%$$

$$U = 10.28 - .005 * 4 * 9.02^2 = 8.653$$

- So the main task is to find Y and remember the following equation and you need to use this equation in your assignment to decide where your Y is for your portfolio

■ To find C, we need three equations:

$$E(r_c) = r_f + y[E(r_p) - r_f]$$

$$\sigma_c^2 = y^2 \sigma_p^2$$

$$U = E(r) - .005 A \sigma^2$$

■ To find C, solve the utility maximization problem

$$\text{Max} U = r_f + y[E(r_p) - r_f] - .005 A y^2 \sigma_p^2$$

$$y^* = \frac{E(r_p) - r_f}{.01 A \sigma_p^2}$$

- Constructing optimal portfolio P, when you find your stocks and put them into portfolio P, what will be the best combination, are you going to put 40 vs 60 % or what is the best weights, how to find P is the key and P is the result of diversification, you can also use passive strategy where P becomes the index funds so the transaction cost will not be very high and administrative fee is also quite low, using passive strategy you can build an index fund and take it as an optimal portfolio and you have risk free assets and all you need is to choose the weights between the two to build the complete portfolio, this is the passive strategy
- Rule no 3, the optimal risk portfolio becomes the final product , now the focus is to find P, now the following equation is to calculate the return of optimal return portfolio P

$$\bar{r}_p = W_1 \bar{r}_1 + W_2 \bar{r}_2 \quad (1)$$

**$W_1$  = Proportion of funds in Security 1**

**$W_2$  = Proportion of funds in Security 2**

**$\bar{r}_1$  = Expected return on Security 1**

**$\bar{r}_2$  = Expected return on Security 2**

$$\sum_{i=1}^n w_i = 1$$

- Use rule no 5 to calculate the risk of optimal portfolio P for two stocks

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \text{Cov}(r_1, r_2) \quad (2)$$

Variance is not a weighted average of the individual asset variances.

$$\sigma_1^2 = \text{Variance of security 1}$$

$$\sigma_2^2 = \text{Variance of security 2}$$

$$\text{Cov}(r_1, r_2) = \text{Covariance of returns for security 1 and security 2}$$

- Use rule no 5 to calculate the risk of optimal portfolio P for three stocks

$$E(r_p) = w_1 E(r_1) + w_2 E(r_2) + w_3 E(r_3)$$

$$\begin{aligned} \sigma_p^2 = & w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + w_3^2 \sigma_3^2 \\ & + 2w_1 w_2 \text{Cov}(r_1, r_2) \\ & + 2w_1 w_3 \text{Cov}(r_1, r_3) \\ & + 2w_2 w_3 \text{Cov}(r_2, r_3) \end{aligned}$$

- Therefore, another way to write the variance of the portfolio is as follows:

$$\sigma_P^2 = W_D W_D \text{Cov}(r_D, r_D) + W_E W_E \text{Cov}(r_E, r_E) + 2W_D W_E \text{Cov}(r_D, r_E)$$

- The variance of the portfolio is a weighted sum of covariances, and each weight is the product of the portfolio proportions of the pair of assets in the covariance term

The variance of a two stock portfolio is the sum of these four boxes

|         | Stock 1                                                          | Stock 2                                                          |
|---------|------------------------------------------------------------------|------------------------------------------------------------------|
| Stock 1 | $x_1^2 \sigma_1^2$                                               | $x_1 x_2 \sigma_{12} =$<br>$x_1 x_2 \rho_{12} \sigma_1 \sigma_2$ |
| Stock 2 | $x_1 x_2 \sigma_{12} =$<br>$x_1 x_2 \rho_{12} \sigma_1 \sigma_2$ | $x_2^2 \sigma_2^2$                                               |

The variance of the portfolio is a weighted sum of covariances

- Add all the above boxes then you get portfolio risk

- Rule no 5,

## Covariance

$$\text{Cov}(r_1 r_2) = \rho_{1,2} \sigma_1 \sigma_2$$

$\sigma_1$  = Standard deviation of returns for security 1

$\sigma_2$  = Standard deviation of returns for security 2

$\rho_{1,2}$  = Correlation coefficient of returns

Therefore,

$$\sigma_p^2 = w_D^2 \sigma_D^2 + w_E^2 \sigma_E^2 + 2w_D w_E \sigma_D \sigma_E \rho_{DE}$$

- How low can portfolio S.D. be? If  $\rho = -1.0$

$$\sigma_P^2 = (W_D \sigma_D - W_E \sigma_E)^2$$

$$W_D \sigma_D - W_E \sigma_E = 0$$

$$W_D = \frac{\sigma_E}{\sigma_D + \sigma_E}$$

$$W_E = \frac{\sigma_D}{\sigma_D + \sigma_E}$$

- $-1.0 \leq \rho \leq +1.0$
  - The smaller the correlation, the greater the risk reduction potential
  - Since correlation coefficient doesn't affect expected return, with other things equal, we always prefer to add assets with lower correlation with our existing position
- To choose stocks you need to look at the returns and risks of 7- 10 stocks, cc, considering the industry diversification effect, once you have chosen your stock use Air New Zealand example on stream and calculate the monthly return, monthly SD, and convert them into

annual return and annual SD then you will end up with something like the table below

## Portfolio Risk and Return

### ■ Example with debt and equity

|                 | Debt | Equity |
|-----------------|------|--------|
| Expected Return | 8%   | 13%    |
| S.D.            | 12%  | 20%    |
| Correlation     | 0.3  |        |

$$E(r_P) = 8W_D + 13W_E$$

$$\sigma_P^2 = 12^2 W_D^2 + 20^2 W_E^2 + 2 * 12 * 20 * .3 * W_D * W_E$$

### ■ We need to find proper weights to get minimum risk portfolio (minimum-variance portfolio)

- But remember this data is only calculating historical information that is the returns and risks are all historical information including CC but in theory when you build portfolios those information returns, risk and CC should be expected values, they should be the value you think will be the next twelve months.
- Now you have the table above, the next stage is to find the P, location of the P depends on the weight of the two stocks, remember we discussed before that the location of C depends on the weight of Y and y is the % you invest in P, eg  $W_D + W_E = 1$ , now the P is on the curve, if P is on point WD it means you you invested 100% of your money in stock D but once you move from this point along the curve towards E then the weight of D will reduce and the weight of E will increase and when it reaches E point then then the weight of E is 100%, so the location of P (optimal risk portfolio) depends on the weight of D and E but the weight of these two are unknown at this point, so if we find the weight then we know where the optimal risk is. You have two equations above and the first equation is rule 3 and the second equation is rule no 5. So where is P, so P is the tangent line touching the curve because at this point that is the tangent point gives the CAL highest sharp ratio, so the slope of the line is the highest i.e the tangent point, why do we want is the highest sharp ratio because the higher the sharp ratio the higher the return, check its equation, so its more efficient. So this P is the tangent point between the CAL and the curve, so how do we find the P, we are trying to maximise the sharp ratio, in the equation below replace the return of the portfolio with rule no 3 then use rule no 5 to replace SD

- With T-bills added in, the optimal risky portfolio is the tangency point that CAL has with the investment opportunity set (the highest feasible reward-to-variability ratio)

$$\underset{W_i}{Max} S_P = \frac{E(r_P) - r_f}{\sigma_P} \quad \sum W_i = 1$$

- 
- in the equation ABOVE replace the return of the portfolio with rule no 3 then use rule no 5 to replace SD to get the following equation

- The weights of the optimal risky portfolio are: (Substitute maximization equation for  $E(r_P)$  from equation (1), and for  $\sigma_P$  from equation (2). Substitute  $1 - W_D$  for  $W_E$ . Differentiate the resulting expression for  $S_P$  with respect to  $W_D$ , set the derivate equal to 0, and solve for  $W_D$ ).

$$W_D = \frac{[E(r_D) - r_f]\sigma_E^2 - [E(r_E) - r_f]Cov(r_D, r_E)}{[E(r_D) - r_f]\sigma_E^2 + [E(r_E) - r_f]\sigma_D^2 - [E(r_D) - r_f + E(r_E) - r_f]Cov(r_D, r_E)}$$

$$W_E = 1 - W_D$$

$$W_D = \frac{(8 - 5)400 - (13 - 5)72}{(8 - 5)400 + (13 - 5)144 - (8 - 5 + 13 - 5)72} = .4$$

$$W_E = 1 - .4 = .6$$

- You can use excel to type the above equation but you need to be very careful but there is a quick way to do this using the functions is excel. So using the equation above you will know the weights of the two stocks then you can find the exact point of P (optimal portfolio)
- Then now you can calculate P and Y using the following equations Now with all the information put together then we can calculate C, now to calculate C you need information of the free risk asset, now we know the P and CAL and with free risk asset information then we can calculate C. And to calculate C we need to calculate Y that is the weight you invest in optimal risk asset then subtract from 1 will the remaing % invested in T Bills  $1 - Y = \text{T Bills}$

- The optimal risky portfolio has 40% in debt and 60% in equity

$$E(r_P) = (.4 * 8) + (.6 * 13) = 11 \%$$

$$\sigma_P = [(.4^2 * 144) + (.6^2 * 400) + (2 * .4 * .6 * 72)]^{\frac{1}{2}} = 14.2 \%$$

$$S_P = \frac{11 - 5}{14.2} = .42$$

- Optimal complete portfolio is given by an optimal risky portfolio, T-bills and investor's risk aversion

$$y = \frac{E(r_P) - r_f}{.01 A \sigma_P^2} = \frac{11 - 5}{.01 * 4 * 14.2^2} = .7439$$

- So investor will invest 74.39% in optimal risky portfolio (P) and 25.61% in T-bills
- P consists of 40% in bonds, and 60% in equity, so the percentage invested in bonds is  $.4 * .7439 = .2976$ , and that in equity is  $.6 * .7439 = .4463$

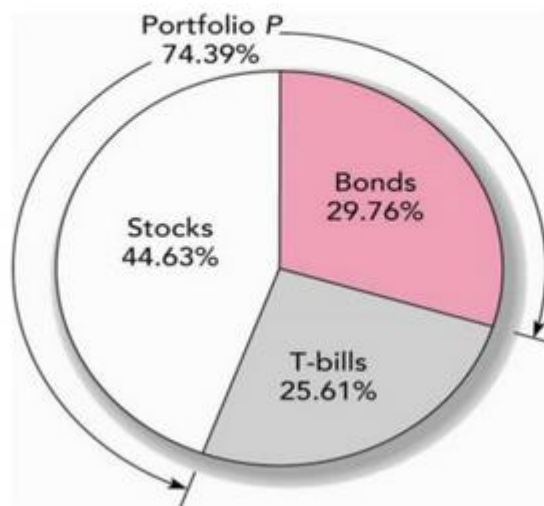
- Now with all the information put together then we can calculate C, now to calculate C you need information of the free risk asset, now we know the P and CAL and with free risk asset information then we can calculate C. And to calculate C we need to calculate Y that is the weight you invest in optimal risk asset then subtract from 1 will the remaining % invested in T Bills  $1 - Y = T$  Bills. But remember that P has two assets, then you can calculate the weights of

each asset using

- So investor will invest 74.39% in optimal risky portfolio (P) and 25.61% in T-bills
- P consists of 40% in bonds, and 60% in equity, so the percentage invested in bonds is  $.4 \times .7439 = .2976$ , and that in equity is  $.6 \times .7439 = .4463$

- Finally your final portfolio will look like this

## ■ The Proportions of the Optimal Overall Portfolio



- Steps in finding the complete portfolio are

## Complete Portfolio

- Specify the return characteristics of all securities (expected returns, variances, covariances)
- Establish the risky portfolio
  - Determine the weights within the optimal risky portfolio
  - Calculate the properties of P using the weights determined in previous step

### THREE STEPS TO FIND COMPLETE PORTFOLIO

#### Step 1 in assignment: Specify the return characteristics of all securities

- Specify the return characteristics of all securities (expected returns, variances, covariances)
- The first thing in your assignment is to construct the following table as it gives you the returns, risk and CC of your securities and to get this table you basically have to use the Air New Zealand example, this table input is step number one

#### Portfolio Risk and Return

- Example with debt and equity

|                 | Debt | Equity |
|-----------------|------|--------|
| Expected Return | 8%   | 13%    |
| S.D.            | 12%  | 20%    |
| Correlation     | 0.3  |        |

$$E(r_p) = 8W_D + 13W_E$$

$$\sigma_p^2 = 12^2 W_D^2 + 20^2 W_E^2 + 2 * 12 * 20 * .3 * W_D * W_E$$

- We need to find proper weights to get minimum risk portfolio (minimum-variance portfolio)

## Step 2 in assignment: Calculate the properties of P

Establish the risky portfolio

- Determine the weights within the optimal risky portfolio
- Calculate the properties of P using the weights determined in previous step

Once you know the returns, risk and CC of the securities, now step no 2 is to find P and to find P you need to calculate the weights of the two securities optimal risk portfolio and once you find the weights together with the information in the table in step 1 above then you can calculate the properties (risk and return) of Optimal risk portfolio. **So step two is to find optimal risk portfolio**

- Specify the return characteristics of all securities (expected returns, variances, covariances)
  - Establish the risky portfolio
    - Determine the weights within the optimal risky portfolio
    - Calculate the properties of P using the weights determined in previous step
- 

## Step 2 in assignment: Find complete portfolio

Allocate funds between the optimal risky portfolio and the risk-free asset

- Calculate the fraction of the complete portfolio allocated to portfolio P and to T-bills
- Calculate the share of the complete portfolio invested in each asset and in T-bills

To find complete portfolio you need to calculate the weights that you invest in optimal risk portfolio, that is Y and then once you know the weights and you know the returns and risk of the two assets then you can calculate the return and risk of the complete portfolio, that is the final product